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CHW solutions

Write your **student number** in the boxes above. **Letter**

Mathematical Methods
Examination 1

Question and Answer Book

VCE Examination – Wednesday 5 November 2025

- Reading time is **15 minutes**: 9.00 am to 9.15 am
- Writing time is **1 hour**: 9.15 am to 10.15 am

Materials supplied

- Question and Answer Book of 16 pages
- Formula Sheet

Instructions

Students are **not** permitted to bring any technology (calculators or software), or notes of any kind, into the examination room.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

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Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- In all questions where a numerical answer is required, an **exact** value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 marks)

a. Let $y = x^2 \cos(x)$.

Find $\frac{dy}{dx}$. 1 mark

$\frac{dy}{dx} = x^2(-\sin(x)) + \cos(x) \cdot 2x$

$= -x^2 \sin(x) + 2x\cos(x)$

b. Let $f(x) = 6\sqrt{x+1} + 5$.

Find the gradient of the tangent to $y = f(x)$ at $x = 8$. 2 marks

$f'(x) = 6 \cdot \frac{1}{2} (x+1)^{-1/2}$

$= \frac{3}{\sqrt{x+1}}$

$f'(8) = \frac{3}{\sqrt{9}} = \frac{3}{3} = 1$

Question 2 (2 marks)

Let $g(x)$ be a function defined for $x > -\frac{3}{2}$ so that $g'(x) = \frac{1}{2x+3}$ and $g(1) = 0$.

Find $g(x)$.

$g(x) = \int \frac{1}{2x+3}\, dx = \frac{1}{2}\log_e(2x+3) + c$, where c is a constant

$g(1) = 0$

$\frac{1}{2}\log_e(5) + c = 0$

$c = -\frac{1}{2}\log_e(5)$

$g(x) = \frac{1}{2}\log_e(2x+3) - \frac{1}{2}\log_e(5)$

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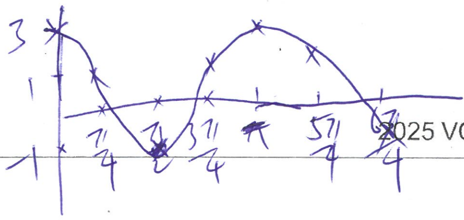


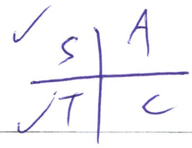
Question 3 (6 marks)

Let $f: [0, 2\pi] \rightarrow R, f(x) = 2\cos(2x) + 1$.

a. State the range of f. 1 mark

$[-1, 3]$

b. Solve $f(x) = 0$ for x. 3 marks



$2\cos(2x) + 1 = 0$

$\cos(2x) = \frac{-1}{2}$

$0 \leq x \leq 2\pi$

$0 \leq 2x \leq 4\pi$

$2x = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}, 3\pi - \frac{\pi}{3}, 3\pi + \frac{\pi}{3}$

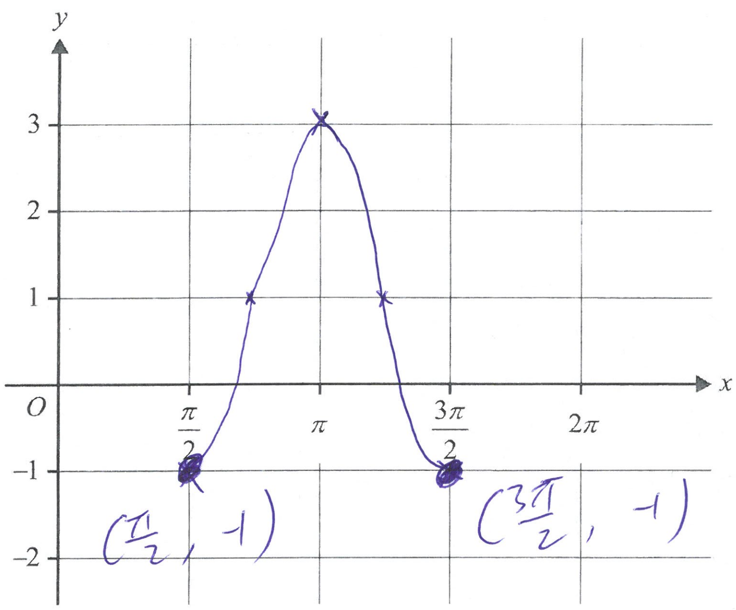
$2x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

c. Sketch the graph of $y = f(x)$ for $x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ on the axes below.

$\frac{6\pi}{4}$

Label the endpoints with their coordinates. 2 marks



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Question 4 (4 marks)

The probability distribution for the discrete random variable X is given in the table below, where k is a positive real number.

$k \quad -15$

$k \quad -10$

$-15k - 10k = -25k$

x	0	1	2	3
$\Pr(X=x)$	$\frac{4}{k}$	$\frac{2k}{75}$	$\frac{k}{75}$	$\frac{2}{k}$

a. Show that $k = 10$ or $k = 15$. 2 marks

$\frac{4}{k} + \frac{2k}{75} + \frac{k}{75} + \frac{2}{k} = 1$

$\frac{300 + 2k^2 + k^2 + 150}{75k} = 1$

$3k^2 + 450 = 75k$

$k^2 - 25k + 150 = 0$

$(k-15)(k-10) = 0$

$k = 15$ or $k = 10$

when $k=15$, ~~[illegible]~~

$0 \le \Pr(X=x) \le 1$ for all x.

when $k=10$,

$0 \le \Pr(X=x) \le 1$ for all x.

Therefore, $k=15$ or $k=10$.

Hopefully VCAA marking scheme includes this.

b. Let $k = 15$.

i. Find $\Pr(X > 1)$. 1 mark

$\frac{15}{75} + \frac{2}{15} = \frac{3}{15} + \frac{2}{15} = \frac{5}{15} = \frac{1}{3}$

ii. Find $E(X)$. 1 mark

$0 \cdot \frac{4}{15} + 1 \cdot \frac{2 \cdot 15}{75} + 2 \cdot \frac{15}{75} + 3 \cdot \frac{2}{15}$

$= \frac{30}{75} + \frac{30}{75} + \frac{6}{15}$

$= \frac{60}{75} + \frac{6}{15} = \frac{12}{15} + \frac{6}{15} = \frac{18}{15} = \frac{6}{5}$

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Question 5 (4 marks)

a. Solve $e^{2x} - 8e^{x} + 7 = 0$ for x. 2 marks

let $a = e^x$

$a^2 - 8a + 7 = 0$

$(a-7)(a-1) = 0$

$e^x = 7$ or $e^x = 1$

$x = \log_e(7)$ or $x = 0$

b. Let $g(x) = e^{2x} - 8e^{x} + 7$, where $x \in R$.

The function $g(x)$ has exactly one stationary point, a local minimum.

Find the largest value of a such that when g is restricted to the domain $(-\infty, a]$ it has an inverse function. 2 marks

$g'(x) = 2e^{2x} - 8e^x$

$g'(x) = 0$

$2e^{2x} - 8e^x = 0$

$e^{2x} - 4e^x = 0$

$e^x(e^x - 4) = 0$

$e^x \neq 0$

$e^x = 4$

$x = \log_e(4)$

$a = \log_e(4)$

Question 6 (3 marks)

Consider the binomial random variable $X \sim \mathrm{Bi}\left(6, \frac{1}{4}\right)$.

a. Find $\mathrm{var}(X)$. 1 mark

$np(1-p) = 6 \cdot \frac{1}{4} \cdot \frac{3}{4}$

$= \frac{3}{2} \cdot \frac{3}{4}$

$= \frac{9}{8}$

b. Determine $\Pr(X \geq 5)$.

Give your answer in the form $\frac{a}{2^b}$, where $a, b \in Z$. 2 marks

$\binom{6}{5}\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^1 + \binom{6}{6}\left(\frac{1}{4}\right)^6\left(\frac{3}{4}\right)^0$

$= 6 \cdot \frac{1}{4^5} \cdot \frac{3}{4} + 1 \cdot \frac{1}{4^6} \cdot 1$

$= \frac{18}{4^6} + \frac{1}{4^6}$

$= \frac{19}{4^6}$

$= \frac{19}{(2^2)^6}$

$= \frac{19}{2^{12}}$

Question 7 (6 marks)

Let $f: R \rightarrow R, f(x) = x^3 - x^2 - 16x - 20$.

a. Verify that $x = 5$ is a solution of $f(x) = 0$. 1 mark

$f(5) = 5^3 - 5^2 - 16 \cdot 5 - 20$

$= 125 - 25 - 80 - 20$

$= 125 - 125$

$= 0$

b. Express $f(x)$ in the form $(x+d)^2(x-5)$, where $d \in R$. 2 marks

$$\begin{array}{r} x^2 + 4x + 4 \\ x-5 \,\big)\, \overline{x^3 - x^2 - 16x - 20} \\ \underline{x^3 - 5x^2} \\ 4x^2 - 16x \\ \underline{4x^2 - 20x} \\ 4x - 20 \\ \underline{4x - 20} \\ 0 \end{array}$$

$f(x) = (x-5)(x^2+4x+4)$

$= (x+2)^2(x-5)$

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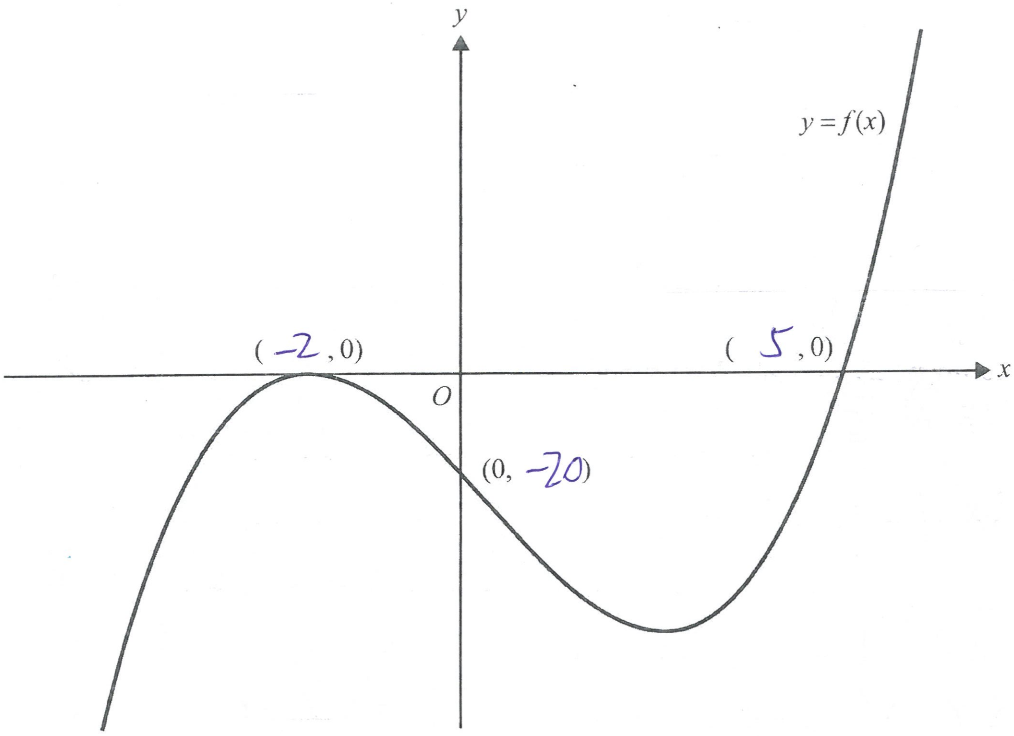
c. Consider the graph of $y = f(x)$, as shown below.

Complete the coordinate pairs of all axial intercepts of $y = f(x)$. 1 mark



d. Let $g : R \to R$, $g(x) = x + 2$.

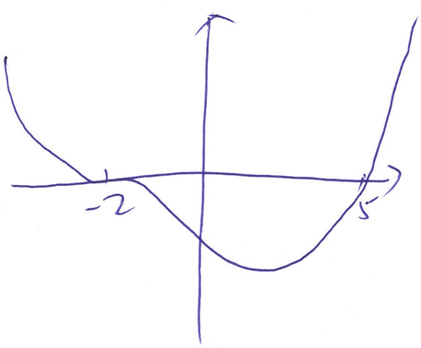
i. State the coordinates of the stationary point of inflection for the graph of $y = f(x)g(x)$. 1 mark

$y = (x+2)^3 (x-5)$

$(-2, 0)$

ii. Write down the values of x for which $f(x)g(x) \geq 0$. 1 mark

$x \in (-\infty, -2] \cup [5, \infty)$



Question 8 (5 marks)

Consider

$$f(x)=\begin{cases}\frac{3}{8}(4-3x) & 0\le x\le \frac{4}{3}\\ 0 & \text{otherwise}\end{cases} \quad = \frac{3}{2} - \frac{9}{8}x$$

$16 \times 3 = 48$

$\frac{24}{9} = \frac{8}{3}$

$3k-7$
$3k-1$
$-3k-21k$

a. The continuous random variable X has probability density function $f(x)$.

Find k such that $\Pr(X>k)=\frac{9}{16}$. 3 marks

$\cancel{\int_k^{4/3} \frac{3}{2} - \frac{9}{8}x \, dx = \frac{9}{16}}$

$\cancel{\left[\frac{3}{2}x - \frac{9}{16}x^2\right]_k^{4/3} = \frac{9}{16}}$

$\frac{3}{8}\int_k^{4/3} 4-3x \, dx = \frac{9}{16}$

$\left[4x - \frac{3}{2}x^2\right]_k^{4/3} = \frac{9}{16}\cdot\frac{8}{3}$

$4\cdot\frac{4}{3} - \frac{3}{2}\left(\frac{4}{3}\right)^2 - 4k + \frac{3}{2}k^2 = \frac{3}{2}$

$\frac{16}{3} - \frac{48}{18} - \frac{3}{2} - 4k + \frac{3}{2}k^2 = 0$

$\frac{8}{3} - \frac{3}{2} - 4k + \frac{3}{2}k^2 = 0$

$16 - 9 - 24k + 9k^2 = 0$

$9k^2 - 24k + 7 = 0$

$(3k-7)(3k-1) = 0$

$k = \frac{7}{3}$ or $k = \frac{1}{3}$

$k = \frac{7}{3}$ is outside domain

$k \neq \frac{7}{3}$

$k = \frac{1}{3}$ only

not enough space?

b. The function $h(x)$ is a transformation of $f(x)$ such that

$$h(x) = mf(x) + n$$

where m and n are real numbers.

Find $\int_0^{\frac{4}{3}} h(x)\,dx$ in terms of m and n. 2 marks

$\int_0^{4/3} \frac{3}{8}m(4-3x) + n \, dx$

$= \frac{3}{8}m\int_0^{4/3} 4-3x \, dx + \int_0^{4/3} n \, dx$

$= \frac{3}{8}m\left[4x - \frac{3}{2}x^2\right]_0^{4/3} + \frac{4}{3}n$

$= \frac{3}{8}m\left[\frac{16}{3} - \frac{3}{2}\left(\frac{4}{3}\right)^2\right] + \frac{4}{3}n$

$= \frac{3}{8}m\cdot\frac{8}{3} + \frac{4}{3}n = m + \frac{4}{3}n$

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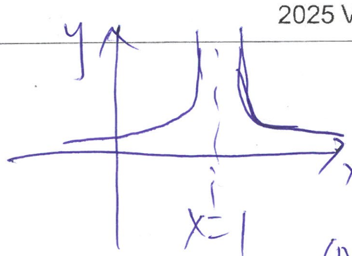
Question 9 (7 marks)

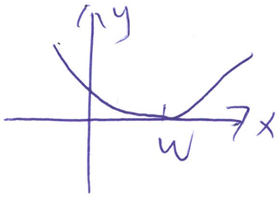
Consider the functions

$$f: R\setminus\{1\} \to R,\ f(x) = \frac{w^2}{(x-1)^2}$$



and

$$g: R \to R,\ g(x) = (x-w)^2$$



where $w \in R$.

a. If $w = -3$, find the four solutions to $f(x) = g(x)$. 3 marks

$\frac{9}{(x-1)^2} = (x+3)^2$

$9 = (x+3)^2(x-1)^2$

$0 = [(x+3)(x-1)]^2 - 3^2$

$0 = [(x+3)(x-1)+3]\cdot[(x+3)(x-1)-3]$

$(x+3)(x-1)+3=0$

$x^2+2x-3+3=0$

$x^2+2x=0$

$x(x+2)=0$

$x=0,\ x=-2$

$(x+3)(x-1)-3=0$

$x^2+2x-3-3=0$

$x^2+2x-6=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-2\pm\sqrt{4-4\cdot1\cdot(-6)}}{2}$

$=\frac{-2\pm\sqrt{28}}{2}$

$=\frac{-2\pm2\sqrt{7}}{2}$

$=-1\pm\sqrt{7}$

$4\times7=28$

$x=0,\ x=-2,\ x=-1+\sqrt{7},\ x=-1-\sqrt{7}$

b. Consider the case where $w > 0$. Absolute ← singular

i. Find, in terms of w, the coordinates of the minimum point of the graph of $y = (x-1)(x-w)$.

2 marks

x-intercepts are $(1, 0)$ and $(w, 0)$

mid-way is $x = \frac{1+w}{2}$. Turning point x-coordinate is the middle of x-values of x-intercepts in parabola.

when $x = \frac{1+w}{2}$, $y = \left(\frac{1+w}{2} - \frac{2}{2}\right)\left(\frac{1+w}{2} - \frac{2w}{2}\right)$

$= \frac{w-1}{2} \cdot \frac{1-w}{2} = \frac{(w-1)(1-w)}{4} = \frac{-(w-1)^2}{4}$

$\left(\frac{1+w}{2}, \frac{-(w-1)^2}{4}\right)$

ii. Hence, or otherwise, find the positive values of w for which $f(x) = g(x)$ has exactly three solutions. ↖ plural

2 marks

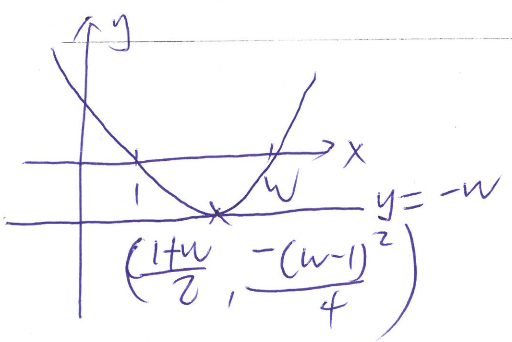
$\frac{w^2}{(x-1)^2} = (x-w)^2$

$w^2 = (x-w)^2(x-1)^2$

$0 = [(x-w)(x-1)]^2 - w^2$

$0 = [(x-w)(x-1) + w] \cdot [(x-w)(x-1) - w]$

$(x-w)(x-1) + w = 0$	$(x-w)(x-1) - w = 0$
$x^2 - x - wx + w + w = 0$	$x^2 - x - wx + w - w = 0$
$x^2 - (1+w)x + 2w = 0$	$x^2 - x - wx = 0$
	$x(x-1-w) = 0$
	$x = 0$ or $x = 1+w$



$-w = \frac{-(w-1)^2}{4}$

$4w = (w-1)^2$

$0 = w^2 - 2w + 1 - 4w$

$0 = w^2 - 6w + 1$

$w = \frac{6 \pm \sqrt{36 - 4 \cdot 1 \cdot 1}}{2}$

$= \frac{6 \pm \sqrt{32}}{2}$

$= \frac{6 \pm 2\sqrt{8}}{2} = 3 \pm \sqrt{8}$

$4 \times 8 = 3$

$w = 3 + 2\sqrt{2}$ or $w = 3 - 2\sqrt{2}$.

End of examination questions