SUPERVISOR TO ATTACH
PROCESSING LABEL HERE

CHW Solutions

Write your **student number** in the boxes above. **Letter**

Mathematical Methods
Examination 2

Question and Answer Book

VCE Examination – Thursday 6 November 2025

- Reading time is **15 minutes**: 11.45 am to 12 noon
- Writing time is **2 hours**: 12 noon to 2.00 pm

Approved materials

- Protractors, set squares and aids for curve sketching
- One bound reference
- One approved CAS calculator or CAS software, and one scientific calculator

Materials supplied

- Question and Answer Book of 28 pages
- Formula Sheet
- Multiple-Choice Answer Sheet

Instructions

- Follow the instructions on your Multiple-Choice Answer Sheet.
- At the end of the examination, place your Multiple-Choice Answer Sheet inside the front cover of this book.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

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Section A – Multiple-choice questions

Instructions

- Answer **all** questions in pencil on your Multiple-Choice Answer Sheet.
- Choose the response that is **correct** for the question.
- A correct answer scores 1; an incorrect answer scores 0.
- Marks will **not** be deducted for incorrect answers.
- No marks will be given if more than one answer is completed for any question.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

A function that has a range of [6, 12] is

A. $f: R \to R, f(x) = 6 + 3\cos(9x)$

B. $f: R \to R, f(x) = 6 + 6\cos(3x)$

C. $f: R \to R, f(x) = 9 - 3\cos(6x)$

D. $f: R \to R, f(x) = 9 - 6\cos(3x)$

Question 2

All asymptotes of the graph of $y = 2\tan\left(\pi\left(x + \frac{1}{2}\right)\right)$ are given by

A. $x = k,\ k \in Z$

B. $x = 2k,\ k \in Z$

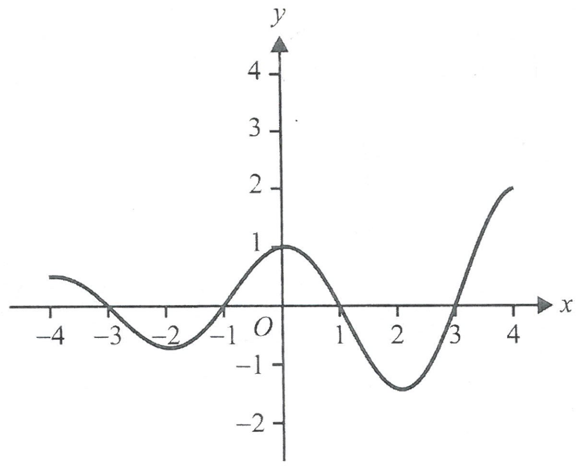
C. $x = 2k + 1,\ k \in Z$

D. $x = \frac{4k+1}{2},\ k \in Z$

UDF
mm_func\asymp

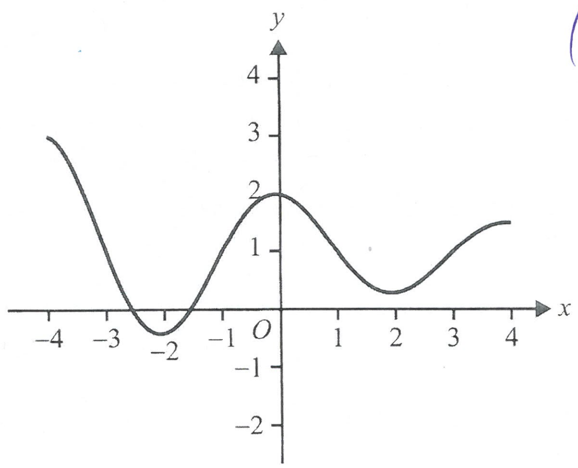
Question 3

The graph of $y = f(x)$ is shown below.



Which one of the following options best represents the graph of $y = f(-x) + 2$?

A.



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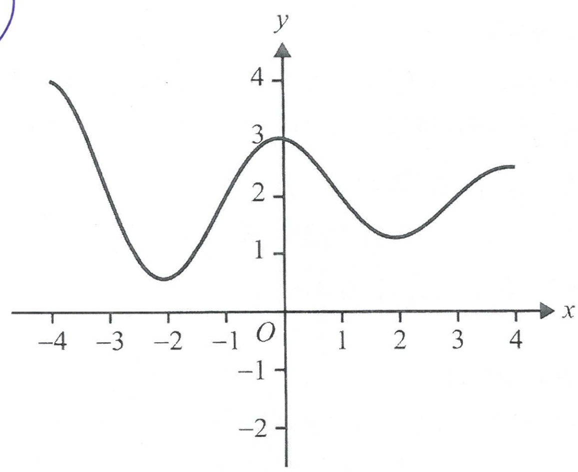

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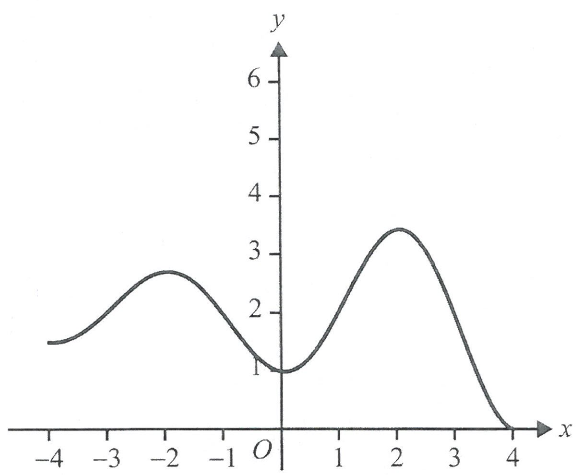
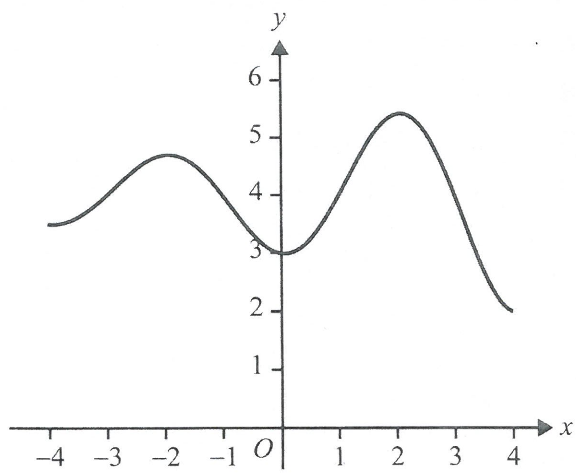


D.



Question 4

Consider the system of equations below containing the parameter k, where $k \in R$.

$$kx + 3y = k^2$$
$$2x + (2k+1)y = 6 - 2k$$

Find the value(s) of k for which this system has no real solutions.

A. $k = -2$ only

B. $k = \frac{3}{2}$ only

C. $k = -2$ or $\frac{3}{2}$

D. $k \in R \setminus \left\{-2, \frac{3}{2}\right\}$

UDF

~~mm~~ mm_func \ linesolve

Question 5

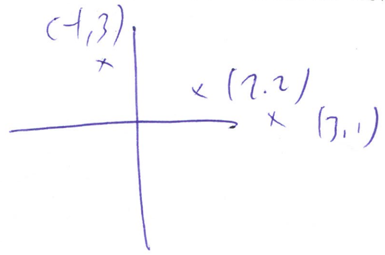
Which of the following sets represents a function that has an inverse function?

A. $\{(1,3), (2,0), (2,1)\}$ ✗

B. $\{(-1,3), (2,2), (3,1)\}$

C. $\{(-1,3), (0,1), (1,3)\}$ ✗

D. $\{(1,0), (2,3), (1,3)\}$ ✗



Question 6

The trapezium rule is used, with two trapeziums, to estimate the area bounded by the graph of $y = f(x)$, the x-axis and the lines $x = 0$ and $x = 1$.

For which function will the trapezium rule estimate be **larger** than the exact area?

A. $f(x) = 3 - e^x$

B. $f(x) = x^3 + 1$

C. $f(x) = 3\sin(x) + 1$

D. $f(x) = \log_e(x+3)$

f(x), x, low, up, 2

Question 7

Consider the algorithm below.

```
n ← 17
k ← 5
while n > k
    n ← n - k
    print n
end while
```

In order, the values printed by the algorithm are

A. 12

B. 12, 7

C. 12, 7, 2

D. 12, 7, 2, −3

Question 8

A random sample of n Victorian households is taken to estimate the proportion of all Victorian households that have vegetable gardens. The approximate 95% confidence interval calculated using this sample is (0.248, 0.552), correct to three decimal places.

The number of households, n, in the sample is

A. 10

B. 28

C. 40

D. 49

$\hat{p} = 0.4 = \frac{0.248 + 0.552}{2}$

$1.96\sqrt{\frac{0.4 \cdot 0.6}{n}} = \frac{0.552 - 0.248}{2}$

$= 0.152$

Question 9

total = m + n

One day, at a particular school, m students walked to school and the remaining n students travelled to school using a different form of transport.

$Pr(30 \text{ min} | m) = 0.2$

Of the m students who walked, 20% took at least 30 minutes to get to school.

Of the n students who used a different form of transport, 40% took at least 30 minutes to get to school.

Given that a randomly selected student took at least 30 minutes to get to school, the probability that they walked to school is given by

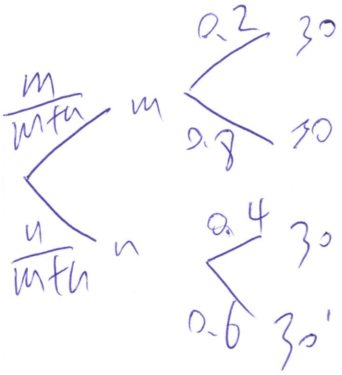
$Pr(30 \text{ min} | n) = 0.4$

A. $\frac{m}{m+2n}$

B. $\frac{2n}{m+2n}$

C. $\frac{m}{5(m+n)}$

D. $\frac{1}{3}$



$Pr(m | 30) = \frac{Pr(m \cap 30)}{Pr(30)}$

$= \frac{\frac{m}{m+n} \cdot 0.2}{\frac{m}{m+n} 0.2 + \frac{n}{m+n} \cdot 0.4}$

Question 10

Consider $f: R \to R,\ f(x) = 2x^2 + x - 1$ and $g: R \to R,\ g(x) = \sin(x)$.

The inequality $(f \circ g)(x) > 0$ is satisfied when

$f \circ g(x) = 2\sin^2(x) + \sin(x) - 1$

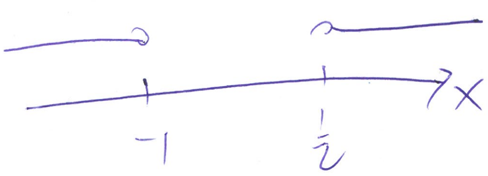
A. $\sin(x) \leq -1$

B. $-1 < \sin(x) < 0$

C. $\frac{1}{2} < \sin(x) \leq 1$

D. $0 < \sin(x) < \frac{1}{2}$

$f(x) > 0$



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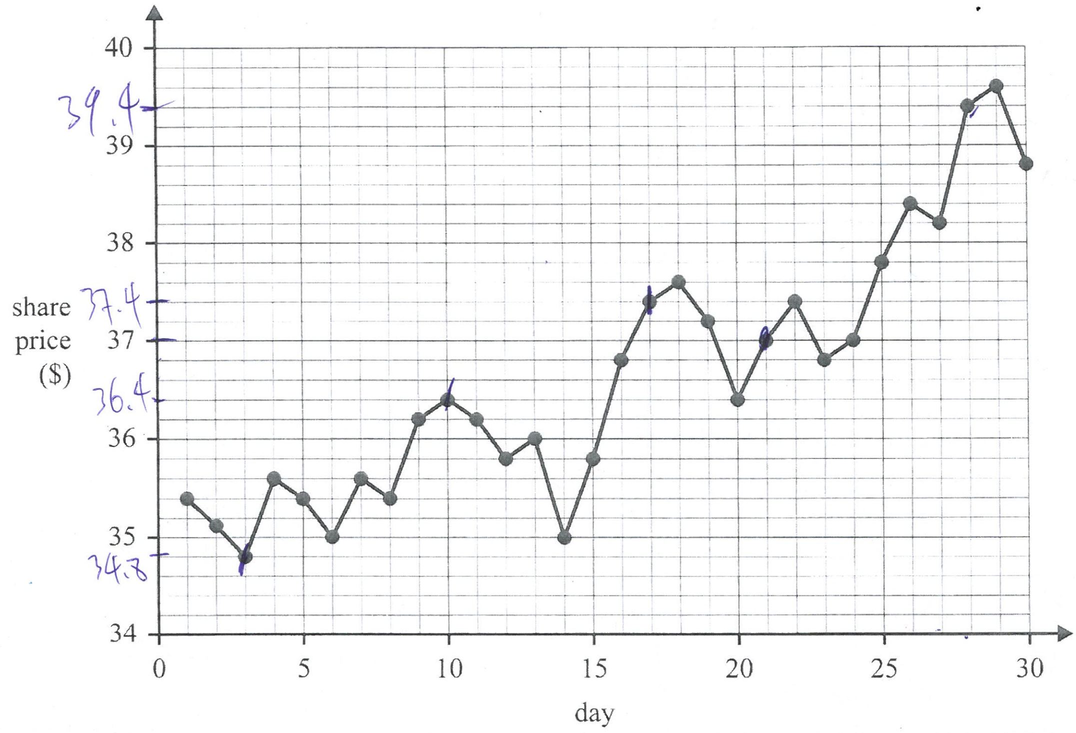
Question 11

The chart below shows the daily price of a stock market share over a 30-day period.



Over which of the following time intervals did the daily price undergo the greatest average rate of change?

A. day 3 to day 10 0.228571

B. day 3 to day 17 0.185714

C. day 14 to day 21 0.285714

D. day 14 to day 28 0.3142

Question 12

For a normal random variable X, it is known that $\Pr(X > 200) = 0.325$ and $\Pr(180 < X < 200) = 0.589$

The mean and standard deviation of X are closest to

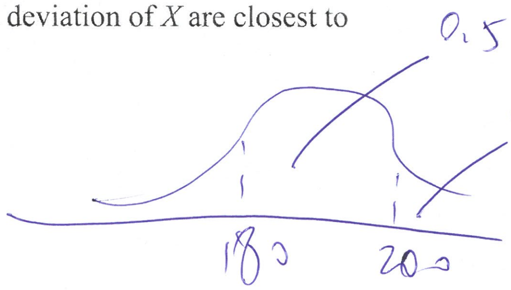
A. 190 and 10

B. 190 and 11

C. 195 and 10

D. 195 and 11

$\Pr(X < 180) = 0.086$



$\Pr\left(Z > \frac{200-\mu}{\sigma}\right) = 0.325$

$\Pr\left(Z < \frac{200-\mu}{\sigma}\right) = 0.675$

$\frac{200-\mu}{\sigma} = 0.453762$

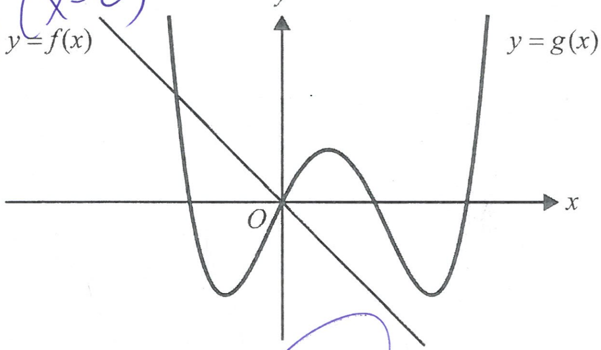
$\Pr\left(Z < \frac{180-\mu}{\sigma}\right) = 0.086$

$\frac{180-\mu}{\sigma} = -1.365806$

Question 13

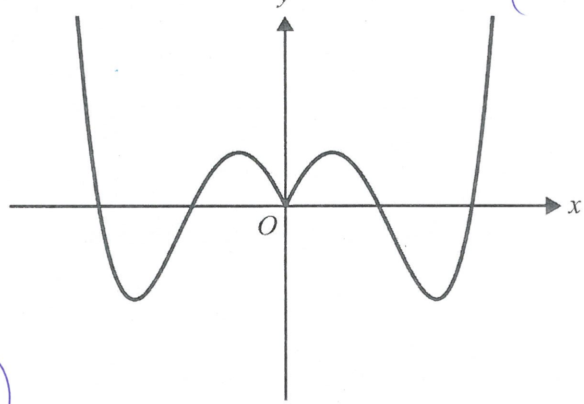
The graphs of $y = f(x)$ and $y = g(x)$ are sketched on the same set of axes below.

$g(x) = (x+1)x(x-1)(x-2)$
$f(x) = -x$



Which of the following could be the graph of $y = (g \circ f)(x)$?

A.



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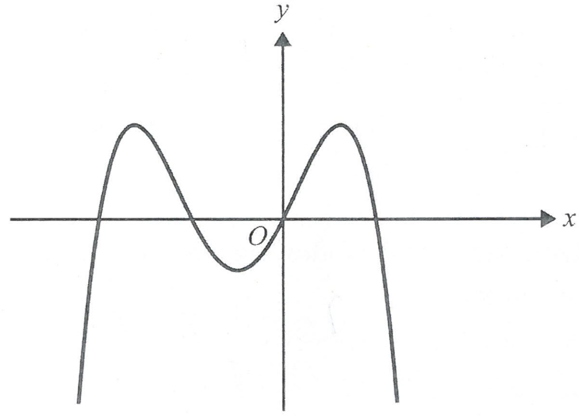
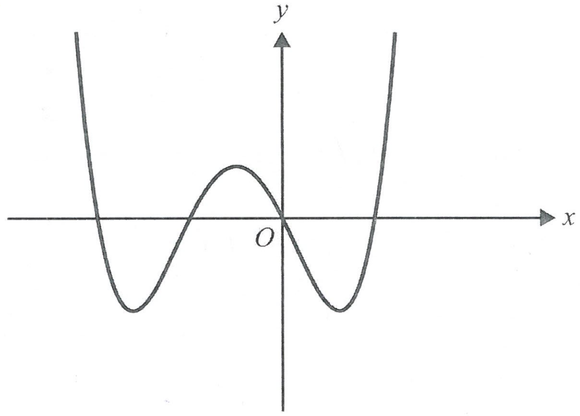
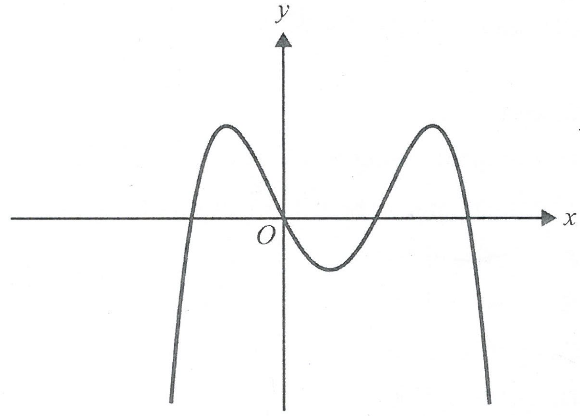


C.



D.



Question 14

Let f be the probability density function for a continuous random variable X, where

$$f(x)=\begin{cases} k\sin(x) & 0 \le x < \frac{\pi}{4} \\ k\cos(x) & \frac{\pi}{4} \le x \le \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$$

and k is a positive real number.

The value of k is

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2-\sqrt{2}}$

C. $\sqrt{2}+2$

D. $2-\sqrt{2}$

$\frac{\sqrt{2}+2}{2} = \frac{\sqrt{2}(1+\sqrt{2})}{\sqrt{2}\sqrt{2}} = \frac{1+\sqrt{2}}{\sqrt{2}}$

1.7071

Question 15

The graph of $y = g(x)$ passes through the point $(1, 3)$.

The graph of $y = 1 - g(2x-3)$ must pass through the point

A. $(-1, -2)$

B. $(2, -2)$

C. $(-1, 2)$

D. $(2, 2)$

$1-g(2(x-\frac{3}{2}))$

$(\frac{1}{2}, -3) \to (2, -2)$

Question 16

Consider the function $h(x) = a\log_e(bx)$, where $a, b \in R \setminus \{0\}$. $b>0$, $x>0$

Given that its derivative $h'(x)$ has range $(0, \infty)$, which of the following **must** be true? $b<0$, $x<0$.

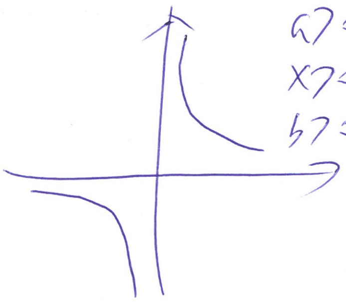
A. $a > 0$ only

B. $a > 0$ and $b < 0$

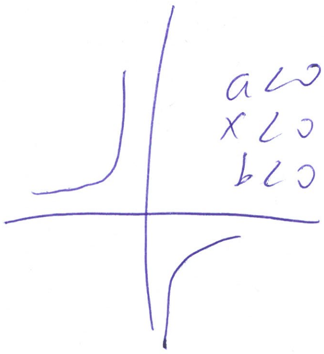
C. ~~$a > 0$ and $b > 0$~~

D. $ab > 0$

$h'(x) = \frac{a}{x}$



$a>0$
$x>0$
$b>0$



$a<0$
$x<0$
$b<0$

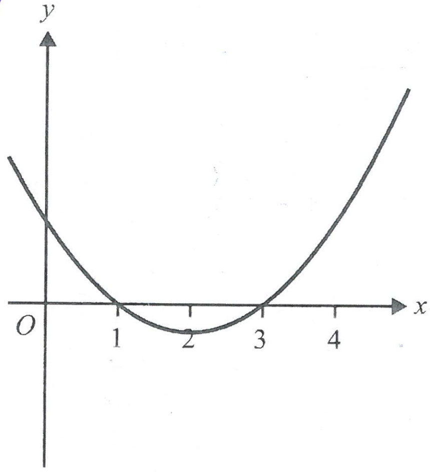
D

Fire

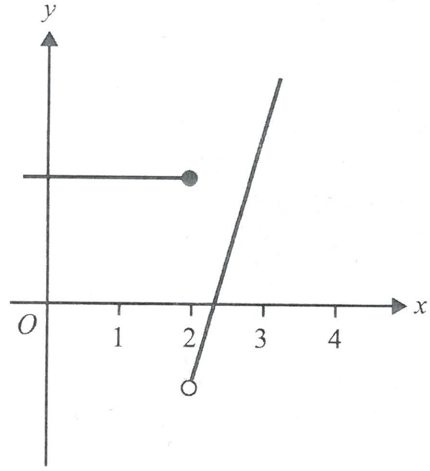
Question 17

Given that $f: R \to R$ satisfies $\int_1^2 f(x)\,dx > \int_1^3 f(x)\,dx$, the graph of $y = f(x)$ could be

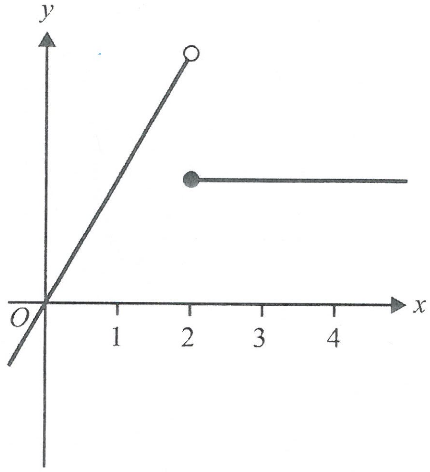
A.



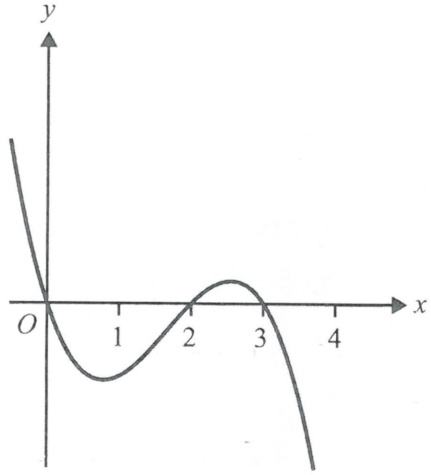
B.



C.



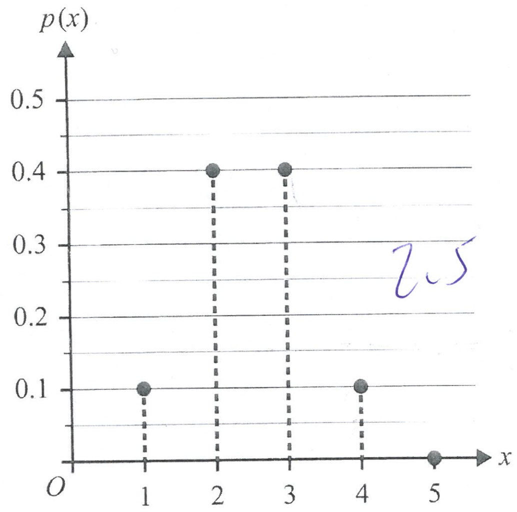
D.



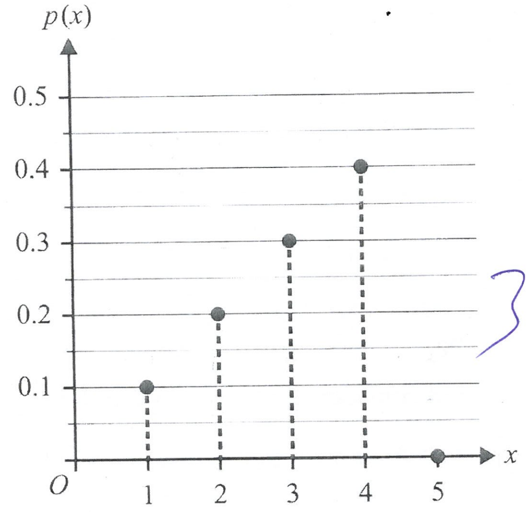
Question 18

Consider the following graphs, which represent probability mass functions.

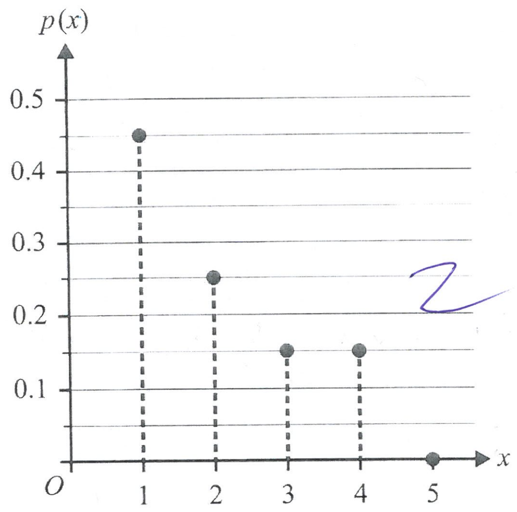
I



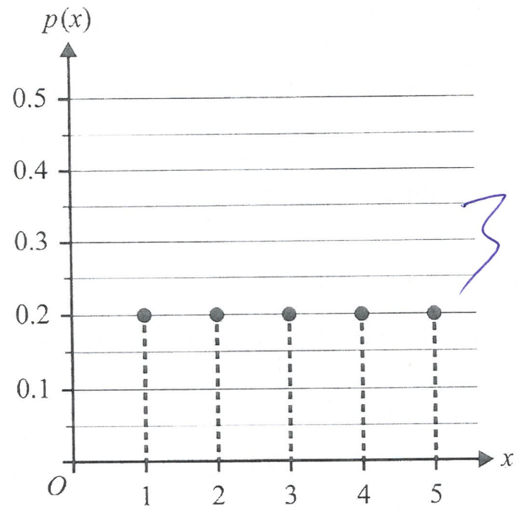
II



III



IV



Which pair of these probability mass functions has the same mean?

A. I and II

B. I and IV

C. II and III

D. II and IV

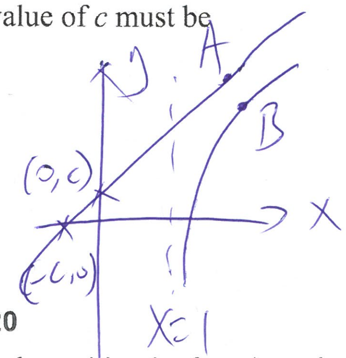
UDF

mm_dpr\ prtable

Question 19

Let A be a point on the line $y = x + c$ and B be a point on the curve $y = \log_e(x-1)$.

If A and B are placed such that the line segment AB has the minimum possible length, and this length is $\sqrt{2}$, the value of c must be



A: $(a, a+c)$ B: $(b, \ln(b-1))$

Normal$_A$: $y = -x + 2a + c$

Normal$_B$: $y = -(b-1)x + \ln(b-1) + b(b-1)$

Same normal $b=2$, $c=c$, $a = \frac{-c+2}{2}$ UDF min func \ point info

A. $\sqrt{2} - 2$

B. $\sqrt{2}$

C. 1

D. 0

Question 20

Let $a > 1$, and consider the functions f and g defined below.

$$f : R \to R,\ f(x) = a^x$$

$$g : R \to R,\ g(x) = a^{2x+2}$$

Which one of the following sequences of transformations, when applied to $f(x)$, does **not** produce $g(x)$?

A. dilation by a factor of $\frac{1}{2}$ from the y-axis, then translation by 1 unit in the negative direction of the x-axis

B. dilation by a factor of $\frac{1}{2}$ from the y-axis, then dilation by a factor of a^2 from the x-axis

C. dilation by a factor of a from the x-axis, then dilation by a factor of $\frac{1}{2}$ from the y-axis, then translation by 1 unit in the positive direction of the x-axis

D. dilation by a factor of a^3 from the x-axis, then translation by 1 unit in the positive direction of the x-axis, then dilation by a factor of $\frac{1}{2}$ from the y-axis

End of Section A

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Examination continues on the next page.

Section B

Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- In questions where a numerical answer is required, an **exact** value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (13 marks)

Let $g: R \to R$ be defined by $g(x) = 4x^3 - 3x^4$.

should be 1 mark only

state

a. Find the coordinates of both stationary points of g. mm_calc\stps **2 marks**

$(0,0)$ $(1,1)$

$g'(x) = 12x^2 - 12x^3$

$g'(x) = 0$

$12x^2 - 12x^3 = 0$

$x^2 - x^3 = 0$

~~$x^2(x - x^2) = 0$~~

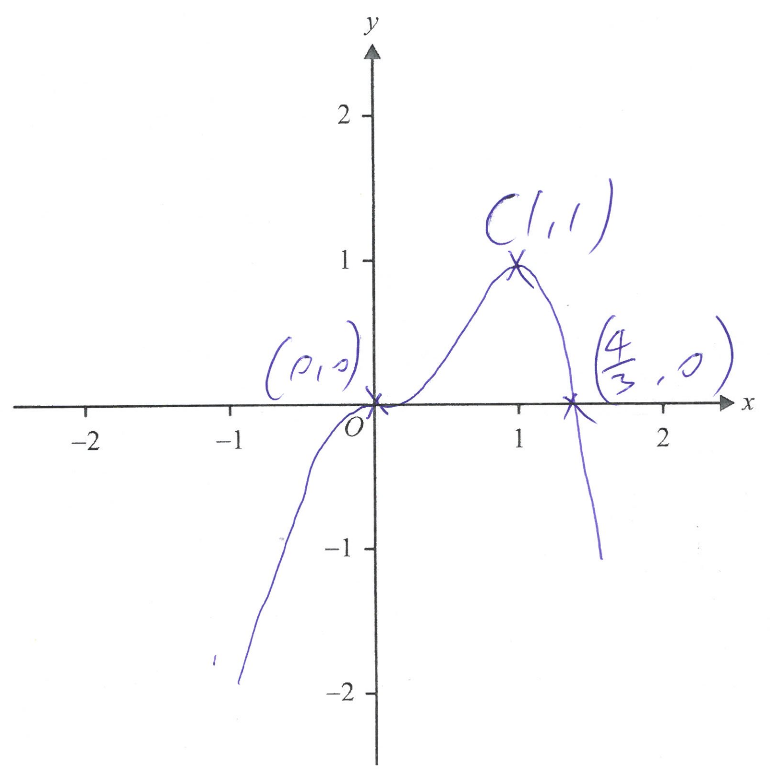
$x^2(1-x) = 0$

$x = 0$ or $x = 1$

$g(0) = 0$

$g(1) = 1$

b. Sketch the graph of $y = g(x)$ on the axes below, labelling the stationary points and axial intercepts with their coordinates. **2 marks**



c. Complete the following gradient table with appropriate values of x and $g'(x)$ to show that g has a stationary point of inflection. 2 marks

x	$-\frac{1}{2}$	0	$\frac{1}{2}$
$g'(x)$	$\frac{9}{2}$	0	$\frac{3}{2}$

d. Find the average value of g between $x = 0$ and $x = 2$. 2 marks

$\frac{1}{2-0}\int_0^2 4x^3 - 3x^4\,dx$

$= \frac{1}{2}\left[x^4 - \frac{3}{5}x^5\right]_0^2$

$= -\frac{8}{5}$

how much working out?

e. Let h be the result after applying a sequence of transformations to g, such that h has a stationary point of inflection at $(1, 0)$ and a local maximum at $(-1, 1)$.

Write down a possible sequence of three transformations to map from g to h. 3 marks

Good question

1. Dilation of factor 2 from the vertical axis
2. Reflection in the vertical axis
3. Translation of 1 unit in the positive direction of the horizontal axis.

f. Let $X \sim \text{Bi}(4, p)$ be a binomial random variable.

Show that $\Pr(X \geq 3) = g(p)$ for all $p \in [0, 1]$. 2 marks

$\Pr(X \geq 3) = \binom{4}{3}p^3(1-p)^1 + \binom{4}{4}p^4(1-p)^0$

$= 4p^3(1-p) + p^4$

$= 4p^3 - 4p^4 + p^4$

$= 4p^3 - 3p^4$

$g(p) = 4p^3 - 3p^4$

$\Pr(X \geq 3) = g(p)$

Question 2 (14 marks)

Let

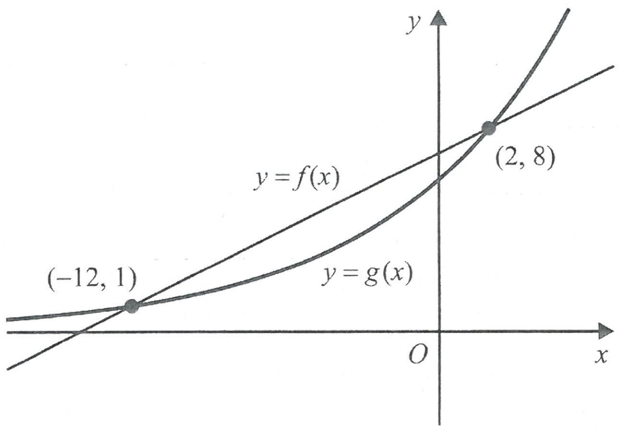
$$f: R \to R,\ f(x) = \frac{x}{2} + 7$$

and

$$g: R \to R,\ g(x) = Ae^{kx}$$

where $A, k \in R$. $A \in R\setminus\{0\},\ k \in R\setminus\{0\}$

The graphs of $y = f(x)$ and $y = g(x)$ intersect at the points $(-12, 1)$ and $(2, 8)$, as shown below.



a. Write down two simultaneous equations in terms of A and k. → exam 1 question

Solve them, using algebra, to show that $A = 2^{\frac{18}{7}}$ and $k = \frac{3}{14}\log_e(2)$. 3 marks

$$\begin{cases} 1 = Ae^{-12k} & (1) \\ 8 = Ae^{2k} & (2) \end{cases}$$

$(1) \times 8$: $8 = 8Ae^{-12k}$ (3)

Between (2) and (3): $Ae^{2k} = 8Ae^{-12k}$

Assume $A \neq 0$

$e^{2k} = 8e^{-12k}$

$e^{14k} = 8$

$14k = \log_e(8) = 3\log_e(2)$

$k = \frac{3}{14}\log_e(2)$

Substitute into (1)

$1 = Ae^{-12 \cdot \frac{3}{14}\ln(2)}$

$1 = A(2)^{-6 \cdot \frac{3}{7}}$

$1 = A\, 2^{-\frac{18}{7}}$

$A = 2^{18/7}$

Do not write in this area.

Do not write in this area.

b. Find the value of b, where $b \in R$, such that $g(x)$ can be expressed in the form $g(x) = A \times 2^{bx}$. → in terms of k? 1 mark

$A \cdot 2^{k / \log_2(e) \cdot x} = Ae^{kx}$

$b = k \log_2(e)$ or $b = \frac{k}{\log_e(2)}$

c. Use a definite integral to evaluate the area bounded by the graphs of $y = f(x)$ and $y = g(x)$, where $x \in [-12, 2]$.

Give the area correct to two decimal places. 2 marks

$\int_{-12}^{2} \frac{x}{2} + 7 - 2^{18/7} e^{3/14 \ln(2) x} \, dx$

$= 15.87$

d. Let $h(x) = f(x) - g(x)$.

i. Write down an expression for the derivative of $h(x)$. → what answer is VCAA looking for? 1 mark

$h'(x) = f'(x) - g'(x)$

$= \frac{1}{2} - \frac{6 \cdot 2^{\frac{3}{14}x + 4/7} \cdot \ln(2)}{7}$

State

ii. Find the maximum value of $h(x)$, where $x \in [-12, 2]$.

Give your answer correct to two decimal places. 1 mark

1.72

e. Let g^{-1} be the inverse of g.

Find the points where the graph of $y = g^{-1}(x)$ intersects with the graph of $y = 2(x - 7)$. 2 marks

$2(x-7)$ is the inverse of $f(x)$, so $f^{-1}(x) = 2(x-7)$

Question is asking for intersections between $y = g^{-1}(x)$ and $y = f^{-1}(x)$. Need to swap x and y-coordinates of intersections between $y = f(x)$ and $y = g(x)$.

$(8, 2)$, $(1, -12)$

Question 2 continues on the next page.

f. Let F be an anti-derivative of f that passes through $(0, c)$, where $c \in R$.

i. Show that it is **not** possible for the graph of $y = F(x)$ to pass through both $(-12, 1)$ and $(2, 8)$. 2 marks

$\int \frac{x}{2} + 7\,dx = \frac{x^2}{4} + 7x + d$ where d is a constant

$c = d$

$F(x) = \frac{x^2}{4} + 7x + c$

~~if $F(-12) = 1$, then~~

if $F(2) = 8$, then

$8 = 1 + 14 + c$

$c = -7$

$F(x) = \frac{x^2}{4} + 7x - 7$

$F(-12) = -55 \neq 1$

So $y = F(x)$ does not pass through $(-12, 1)$.

So not possible to pass through both points.

ii. The graph of $y = F(x)$ can be dilated by a factor of m from the x-axis such that its image passes through both $(-12, 1)$ and $(2, 8)$.

Find the values of m and c. 2 marks

Let the dilated function be $F^*(x)$

$F^*(x) = m\left(\frac{x^2}{4} + 7x + c\right)$

$\begin{cases} F^*(-12) = 1 \\ F^*(2) = 8 \end{cases}$

$m = \frac{1}{9}$ and $c = 57$

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Question 3 (14 marks)

The time taken for a driver to travel to work each day, in minutes, is modelled by a continuous random variable T with probability density function

$$f(t)=\begin{cases}\frac{1}{1215000}(t-29)(59-t)^3 & 29\le t\le 59\\ 0 & \text{otherwise}\end{cases}$$

a. i. Find the mean time taken, in minutes, for the driver to travel to work each day. 1 mark

39

ii. Find the standard deviation of the time taken, in minutes, for the driver to travel to work each day. 2 marks

$\sigma^2 = \int_{29}^{59} (t-39)^2 f(t)\,dt$

$= \frac{200}{7}$

$\sigma = \sqrt{\frac{200}{7}} = \frac{10\sqrt{14}}{7}$

b. The driver allows k minutes to travel to work each day. If the journey takes longer than k minutes, the driver will be late. Whether the driver is late on a particular day is independent of whether they are late on any other day.

i. If $k = 47$, write a definite integral to show that the probability of the driver being late is 0.08704 — 1 mark

$\int_{47}^{59} f(t)\,dt = 0.08704$

ii. If $k = 47$, find the probability that the driver will be late on at least one day in a five-day working week.

Give your answer correct to four decimal places. — 2 marks

let $X \sim Bi(5, 0.08704)$

$Pr(X \geq 1) = 0.3658$

iii. For $k = 47$, let $\hat{P}$ be the proportion of days the driver is late in any five-day working week. Find $\Pr(0.4 \leq \hat{P} \leq 0.6)$ correct to four decimal places. — 2 marks

$\hat{p} = 0.4 = \frac{2}{5} \Rightarrow x = 2$

$\hat{p} = 0.6 = \frac{3}{5} \Rightarrow x = 3$

$Pr(0.4 \leq \hat{p} \leq 0.6) = Pr(2 \leq x \leq 3) = 0.0631$

iv. Find the **integer** k such that the probability, correct to one decimal place, of the driver being late at least once in any five-day working week is 0.2 — 2 marks

let $\int_{k}^{59} f(t)\,dt = p$

~~let~~ let $Y \sim Bi(5, p)$

$Pr(Y \geq 1) = 1 - Pr(Y = 0) = 1 - \binom{5}{0} p^0 (1-p)^5$

$= 1 - (1-p)^5 = 0.2$

$p = 0.043648$

$\int_{k}^{59} f(t)\,dt = 0.043648$

$k = 49$

Question 3 continues on the next page.

c. At a given traffic light, the wait time is modelled by a normal distribution with a mean of 2.5 minutes and a standard deviation of σ minutes. $\mu = 2.5$, $\sigma = ?$

i. If $\sigma = 0.6$, find the probability that the wait time will be less than 3.5 minutes. <3.5

Give your answer correct to two decimal places. 1 mark

0.95

0.02

ii. Find the value of σ such that there is a 2% chance of a wait time longer than 3.5 minutes. >3.5

Give your answer correct to two decimal places. 1 mark

let $X \sim N(2.5, \sigma^2)$

$\Pr(X > 3.5) = 0.02$

$\Pr(X < 3.5) = 0.98$

let Z be standard normal variable

$\Pr\left(Z < \frac{3.5-2.5}{\sigma}\right) = 0.98$

$\frac{1}{\sigma} = 2.053749$

$\sigma = 0.49$

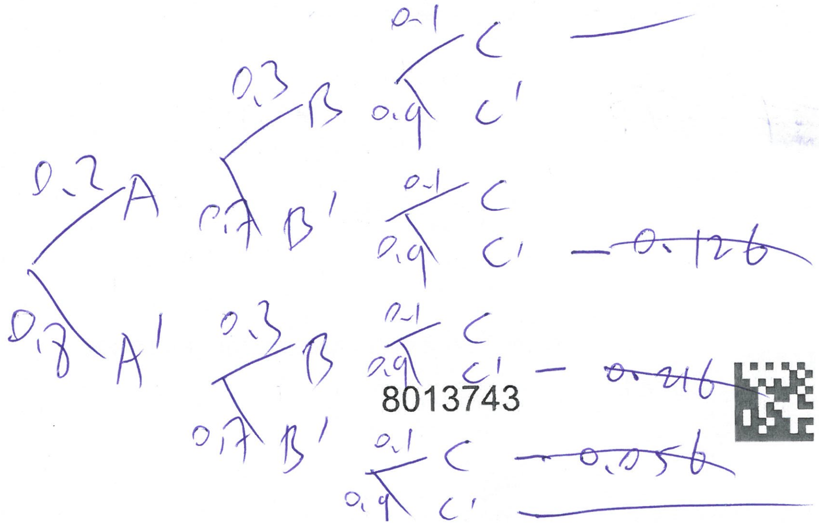
d. The driver passes through three traffic lights (A, B and C) on their journey to work. The probability of each traffic light being red is shown in the table below.

Traffic light	A	B	C
Probability that the traffic light is red	0.2	0.3	0.1

Let Y be the random variable representing the number of traffic lights that are red on the driver's journey to work. Assume that each traffic light being red is independent of any other traffic light being red.

Complete the following table for the probability distribution of Y. 2 marks

y	0	1	2	3
$\Pr(Y = y)$	0.504	0.398	0.092	0.006



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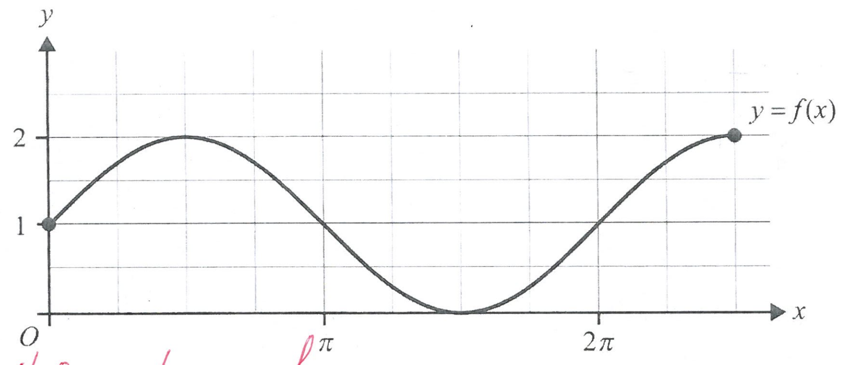
Question 4 (19 marks)

Consider the function $f:\left[0, \frac{5\pi}{2}\right] \to R,\ f(x) = \sin(x) + 1$.

The graph of $y = f(x)$ is shown below.



State the value of

a. Evaluate $f\left(\frac{2\pi}{3}\right)$. 1 mark

$\frac{\sqrt{3}}{2} + 1$

State

b. Find the exact values of x for which $f(x) = \frac{3}{2}$. 1 mark

$x = \frac{\pi}{6}$, $x = \frac{5\pi}{6}$, $x = \frac{13\pi}{6}$

plural exclude

c. There exist real numbers a and k in the interval $\left(0, \frac{5\pi}{2}\right)$, such that $f(x+k) = f(x)$ for all $x \in [0, a]$.

good question

singular

Find the value of k and the largest possible value of a. 2 marks

$K = 2\pi$

~~Domain of f is $[0, \frac{5\pi}{2}]$, so it makes sense that $a = \frac{5\pi}{2}$, but a is not allowed to be $\frac{5\pi}{2}$?~~

~~$0 < x < \frac{5\pi}{2}$~~

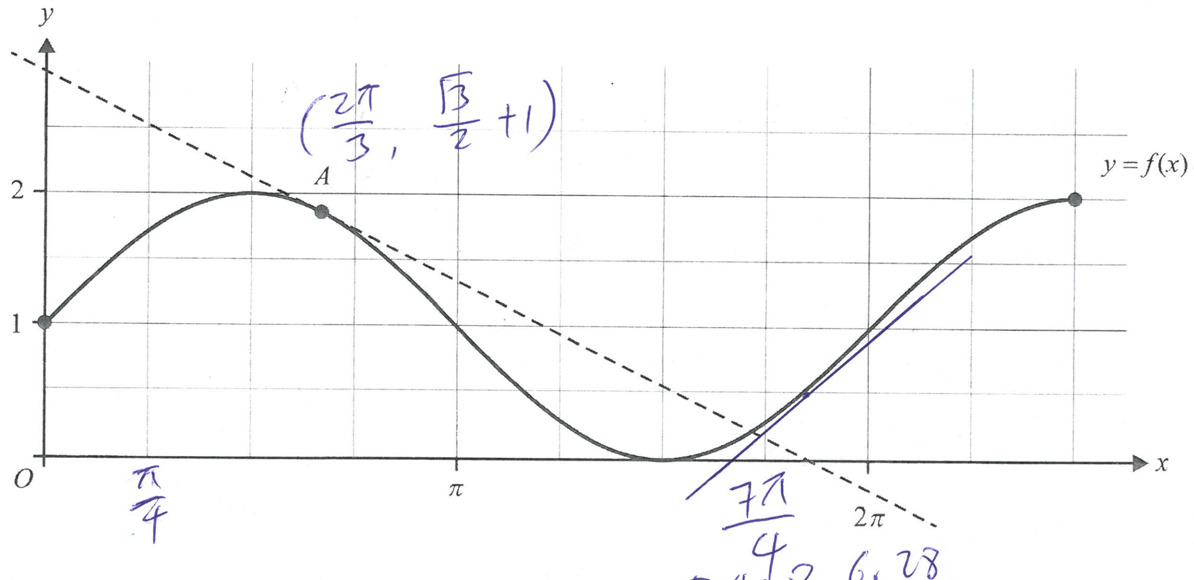
~~$2\pi < x + 2\pi <$~~

$f(x+2\pi)$ has domain $[-2\pi, \frac{\pi}{2}]$

$a = \frac{\pi}{2}$

Question 4 continues on the next page.

d. Consider the tangent to the graph of $y = f(x)$ at the point A where $x = \frac{2\pi}{3}$, as shown on the axes below.



Find the equation of the tangent to the graph of $y = f(x)$ at the point where $x = \frac{2\pi}{3}$. 1 mark

$y = \frac{-x}{2} + \frac{2\pi + 3(\sqrt{3}+2)}{6}$

mm_calc\newton(ex, var, x0, n)

e. Apply two iterations of Newton's method to f with $x_0 = \frac{2\pi}{3}$.

i. Write down x_2, correct to one decimal place. 1 mark

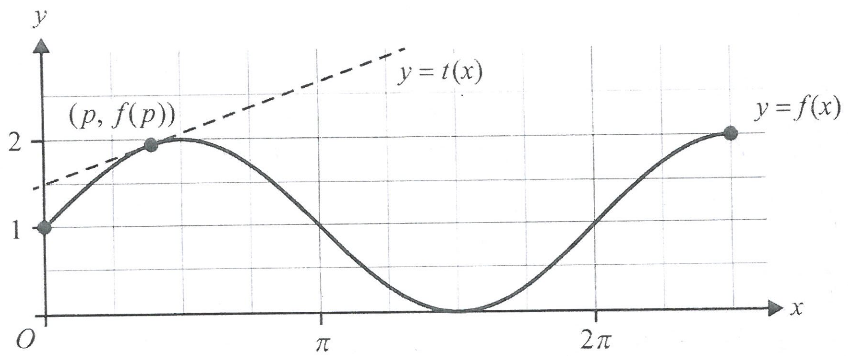
5.2

ii. On the axes in **part d**, draw the tangent to the graph of $y = f(x)$ at the point where $x = x_1$. 1 mark

$x_1 = 5.826$

*(Answer on the graph in **part d**.)*

f. Now consider the line $y = t(x)$, which is the tangent to the graph of $y = f(x)$ at the point $(p, f(p))$, where $p \in \left(0, \frac{5\pi}{2}\right)$.



i. Show that $t(x) = \cos(p)(x-p) + \sin(p) + 1$. 2 marks

$f'(x) = \cos(x)$
$f'(p) = \cos(p)$
$t(x) = \cos(p) \cdot x + c$
$f(p) = \sin(p) + 1$
$t(p) = f(p)$
$\cos(p) \cdot p + c = \sin(p) + 1$
$c = \sin(p) + 1 - \cos(p) \cdot p$
$t(x) = \cos(p) \cdot x + \sin(p) + 1 - \cos(p)p$
$t(x) = \cos(p)(x-p) + \sin(p) + 1$

ii. Determine the minimum and maximum possible values for the y-coordinate of the ~~y~~-intercept of $y = t(x)$, for $p \in \left(0, \frac{5\pi}{2}\right)$. 2 marks

$t(0) = -p\cos(p) + \sin(p) + 1$
min: $p = 2\pi$, max: $p = \pi$
Min is $(1 - 2\pi)$, Max is $(\pi + 1)$

iii. Determine the values plural of p for which $y = t(x)$ has a unique x-intercept that is equal to the x-intercept of $y = f(x)$.

Give your answers correct to two decimal places. 2 marks

$0 = f(x) \Rightarrow x = \frac{3\pi}{2}$
$0 = t(x) \Rightarrow x = \frac{p\cos(p) - \sin(p) - 1}{\cos(p)}$
$\frac{3\pi}{2} = \frac{p\cos(p) - \sin(p) - 1}{\cos(p)}$
$p = 2.38$ or $p = 7.04$

Question 4 continues on the next page.

g. Let $g:\left[0, \frac{5\pi}{2}\right] \to R$, $g(x) = ax^3 + bx^2 + cx + d$ be a polynomial function,

where $a, b, c, d \in R$.

Suppose $g(0) = f(0)$ and $g'(0) = f'(0)$. $f'(x)$ is not defined at $x=0$

i. Show that $c = 1$ and $d = 1$. 2 marks

From $g(0)=f(0)$

$d = \sin(0)+1 = 1$

$f'(x) = \cos(x)$

$g'(x) = 3ax^2+2bx+c$

From $g'(0)=f'(0)$

$c = \cos(0) = 1$

ii. If $g(2\pi) = f(2\pi)$ and $g'(2\pi) = f'(2\pi)$, determine the area bounded by the graphs of $y = f(x)$ and $y = g(x)$, for $x \in [0, 2\pi]$.

Give your answer correct to two decimal places. 2 marks

Solving $\begin{cases} g(2\pi)=f(2\pi) \\ g'(2\pi)=f'(2\pi) \end{cases}$ gives $a=\frac{1}{2\pi^2}$ and $b=\frac{-3}{2\pi}$

$g(x) = \frac{1}{2\pi^2}x^3 - \frac{3}{2\pi}x^2+x+1$

$f(x)=g(x) \Rightarrow x=\pi$

$\int_0^{\pi} f(x)-g(x)\,dx + \int_{\pi}^{2\pi} g(x)-f(x)\,dx$

$= 1.53$

iii. Let $a = 0$, $c = 1$, $d = 1$.

Find b and r, such that $g(r) = f(r)$ and $g'(r) = f'(r)$, where $b \in R$ and $r \in \left(0, \frac{5\pi}{2}\right)$. 2 marks

$g(x) = bx^2+x+1$

$r = \pi$

$b = \frac{-1}{\pi}$

End of examination questions

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