bigWU

VCAA VCE Mathematical Methods Units 3 & 4

Written Examination 2 (Technology-active) 2026

Question and Answer Book

Student Name ________________

Teacher Name ________________

Reading Time: 15 minutes
Writing Time: 120 minutes

The total number of marks available is 80

Approved Materials
- Protractors, set squares, and aids for curve sketching
- One bound reference
- One approved CAS calculator or CAS software, and one scientific calculator

Materials Supplied
- Question and Answer Book
- Formula Sheet
- Multiple-choice Answer Sheet

Instructions
- Follow the instructions on your Multiple-choice Answer Sheet.
- At the end of the examination, place your Multiple-choice Answer Sheet inside the front cover of this book.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Section A – Multiple-choice Questions

Instructions

- Answer all questions in pencil on your Multiple-choice Answer Sheet.
- Choose the response that is correct for the question.
- A correct answer scores 1; an incorrect answer scores 0.
- Marks will not be deducted for incorrect answers.
- No marks will be given if more than one answer is completed for any question.
- Unless otherwise indicated, the diagrams in this book are not drawn to scale.

The following algorithm written in pseudocode (***Algorithm 1***) applies to **Question 1** and **Question 2**. It compares two approximations of the derivative of the function $f(x) = x^2 - 2x + 1$, one of which is the central difference approximation.

```
define f(x):
      return x^2 − 2x + 1
h ← 0.5
a ← 2
m ← 2
i ← 1
f1 ← 1
f2 ← 1
while (f1 − m) < −0.01 or (f1 − m) > 0.01 or (f2 − m) < −0.01 or (f2 − m) > 0.01
      f1 ← (f(a+h) − f(a−h)) / (2h)
      f2 ← (f(a+h) − f(a)) / h
      print(i,h,f1,f2)
      i ← i + 1
      h ← h/2
end while
```

Algorithm 1

Question 1

When ***Algorithm 1*** is run, in the <u>first</u> line of output, what will be the values of $f1$ and $f2$?

A. 2.25 and 2.5 respectively
B. 2.0 and 2.5 respectively
C. 3.0 and 2.0 respectively
D. 3.35 and 3.2 respectively

Question 2

When ***Algorithm 1*** is run, in the <u>final</u> line of output, what will be the value of i?

A. 9
B. 8
C. 11
D. 10

Question 3

The following algorithm written in pseudocode estimates the area between the graph of $y = f(x)$ and the x-axis using trapeziums.

```
define f(x):
      return x^2 − 2x + 1
sum ← 0
a ← 0
b ← 3
n ← 6
[redacted]
left ← a
right ← a + h
for i from 1 to n
      strip ← 0.5 × (f(left) + f(right)) × h
      sum ← sum + strip
      left ← left + h
      right ← right + h
end for
print sum
```

One line of code is missing and blacked out. What should this line of code be?

A. $h \leftarrow \frac{b-a}{n}$

B. $h \leftarrow 1$

C. $h \leftarrow \frac{b+a}{n}$

D. $h \leftarrow \frac{a+1}{b}$

Question 4

What is the result of evaluating $\int_{-\frac{5}{2}}^{\frac{5}{2}} \frac{1}{25-x^2}\, dx$?

A. $\frac{1}{10}\log_e(4)$

B. $\frac{1}{2}\log_e(2)$

C. $\frac{1}{3}\log_e(5)$

D. $\frac{1}{5}\log_e(3)$

Question 5

Consider function $g: (1, \infty) \to \mathbb{R}, g(x) = \frac{1}{x^2} + 2x + 3$. What is the rule of the asymptote in the graph of $y = g(x)$?

A. $y = x$

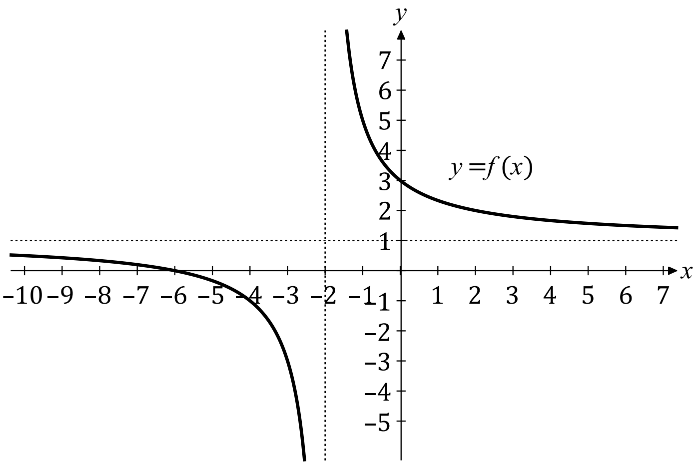
B. $y = 2x + 3$

C. $y = 0$

D. $y = 2x$

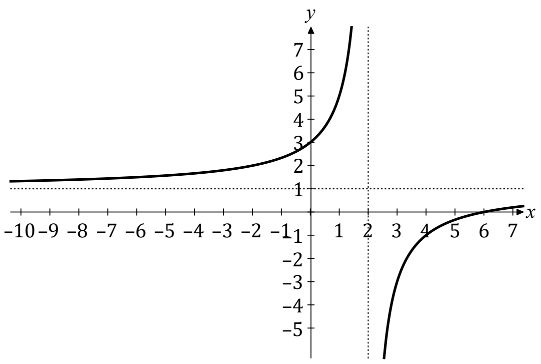
Question 6

The diagram below shows the graph of $y = f(x)$, where function f is a rectangular hyperbola.

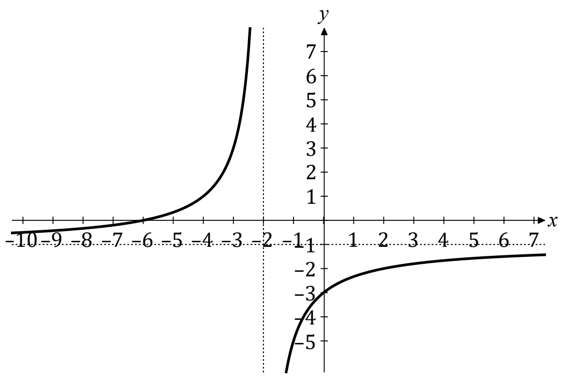


What is the graph of $y = f\left(\frac{1}{x}\right)$?

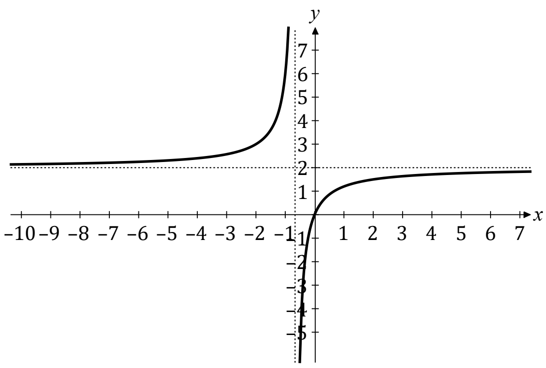
A.



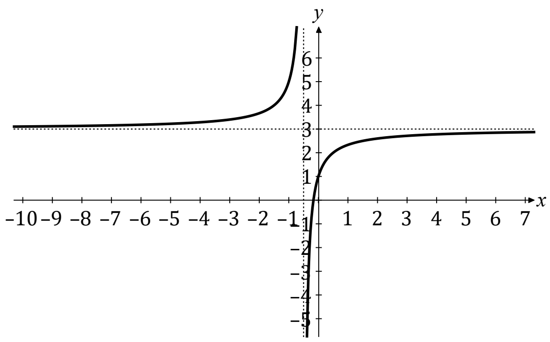
B.



C.



D.



Question 7

The graph of $y = f(x)$ is translated by 4 units in the positive direction of the y-axis, reflected in the x-axis, reflected in the y-axis, then translated by 2 units in the positive direction of the x-axis, to obtain the graph of $y = g(x)$, where $g(x) = (x-5)(x-2)^2$. The rule of $f(x)$ is

A. $(x+1)(x-2)^2$

B. $(x+1)^2(x-2)$

C. $(x-1)(x+2)^2$

D. $(x-1)^2(x+2)$

Question 8

The amount of time, X minutes, that Henry spends each night on Maths homework is a continuous random variable and its probability density function is $f(x)$, where $f(x) = 0$ for $x < 0$ and $f(x) \neq 0$ for $x \geq 0$. The cumulative distribution function of X is $F(x) = 1 - \frac{1}{x+1}$. The rule of $f(x)$ for $x \geq 0$ is

A. $-\frac{1}{(x+1)^2}$

B. $\frac{1}{x+1}$

C. $-\frac{1}{x+1}$

D. $\frac{1}{(x+1)^2}$

The graph of $y = f(x)$, which consists of two connected line segments, is shown in ***Diagram 1*** below, where $0 < a < b < c$. ***Diagram 1*** applies to **Question 9** and **Question 10**.

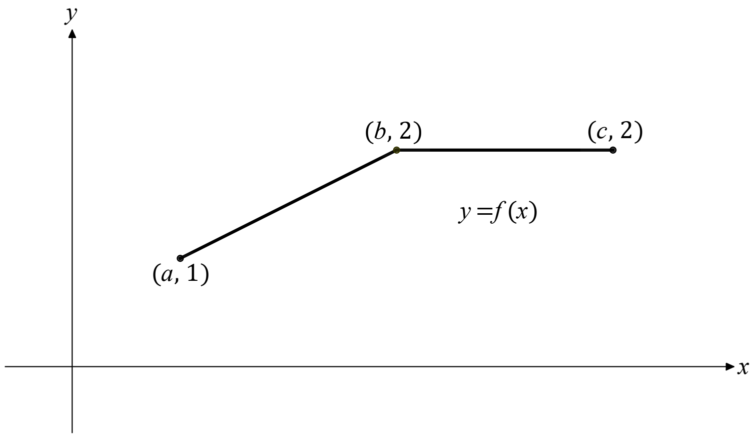


Diagram 1

Question 9

What is the average rate of change of f for $a \leq x \leq c$, where the graph of $y = f(x)$ is shown in ***Diagram 1*** above?

A. $\frac{1}{c-a}$

B. 0

C. $\frac{2}{b-a}$

D. $\frac{1}{b-a}$

Question 10

What is the average value of f over the interval $[a, c]$, where the graph of $y = f(x)$ is shown in ***Diagram 1*** above?

A. $\frac{4c-b-3a}{2c-2a}$

B. $\frac{4}{c-a}$

C. $\frac{b-3a+2c}{a-c}$

D. $\frac{4c-b-3a}{2}$

Question 11

A discrete random variable, X has a probability distribution described by

$$\Pr(X = x) = \frac{ax}{4}$$

where $x \in \{1,2,3\}$ and a is a real constant.

What is the value of a?

A. 1

B. $\frac{2}{3}$

C. $\frac{1}{3}$

D. $\frac{4}{5}$

Question 12

Consider function $f: (0, a] \to \mathbb{R}, f(x) = \log_e(x)\sin(2x)$. What is the largest set of values of a for which f has an inverse function, correct to four decimal places?

A. $a \in (0, 0.3145]$

B. $a \in (0, 0.3145)$

C. $a \in [0, 0.3145]$

D. $a \in [0, 0.3145)$

Question 13

In rolling an <u>uneven</u> six-sided die, the numbers 2 to 6 inclusive have an equal probability of being obtained. The probability of rolling a 1 is greater than the probability of rolling any other number. When the die is rolled 152 times, the number 1 is obtained 57 times. By using the experimental frequency of rolling a 1, what is the probability of rolling a 6?

A. $\frac{1}{11}$

B. $\frac{1}{8}$

C. $\frac{1}{10}$

D. $\frac{1}{9}$

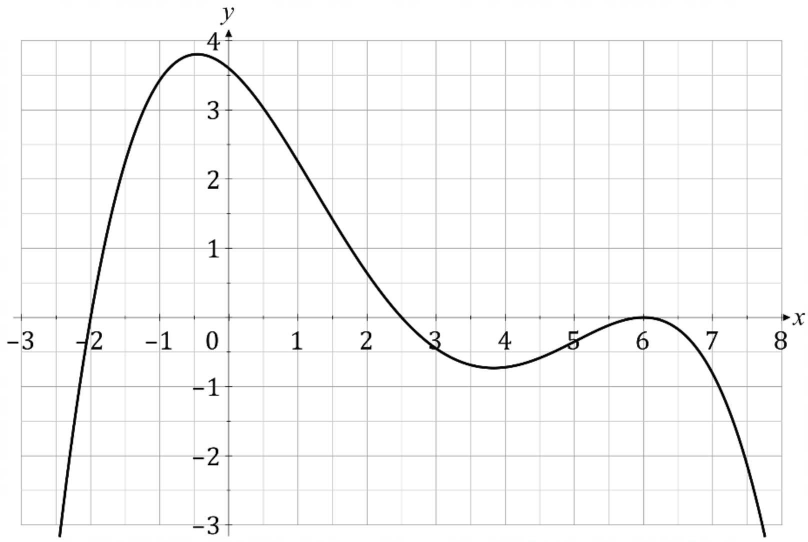
Question 14

The minimum daily temperature, in °C, of a city is recorded for a year. The data follows a normal distribution with the mean and the standard deviation being equal to each other. What is the percentage of days in the year where the minimum daily temperature is above 0°C?

A. 84%
B. 16%
C. 68%
D. 50%

Question 15

The diagram below shows the graph of $y = f'(x)$.



Given $f(1) = 7$, which interval includes the best estimate for $f(1.1)$?

A. $(8.0, 8.5)$
B. $(7.0, 7.2)$
C. $(6.8, 7.0)$
D. $(6.6, 6.8)$

Question 16

A discrete random variable X has probability distribution

$$\Pr(X = x) = \frac{x+1}{3} \text{ where } x \in \{0,1\}.$$

What are the mean and variance of X?

A. $E(X) = \frac{1}{3}, Var(X) = \frac{2}{9}$
B. $E(X) = \frac{1}{3}, Var(X) = \frac{2}{3}$
C. $E(X) = \frac{2}{3}, Var(X) = \frac{2}{9}$
D. $E(X) = \frac{2}{3}, Var(X) = \frac{2}{3}$

Question 17

Given that a is a constant and $a \in \mathbb{Z}^+$, how many distinct solutions are there to the equation $\cos(ax) + \sin(x) = 0$ for $0 \leq x \leq 2\pi$?

A. $3a - 1$

B. $3a$ if a is even, $3a - 1$ if a is odd

C. $2a$ if a is odd, $2a - 1$ if a is even

D. $2a$ if a is even, $2a - 1$ if a is odd

Question 18

For a function $f(x)$, it is known that $f(3) = 1$, $f'(3) = 2$, and $f''(3) = 4$.
Let $g(x) = f^{-1}(x)$. What is the value of $g''(1)$?

A. $-\frac{1}{4}$

B. $\frac{1}{4}$

C. -1

D. $-\frac{1}{2}$

The graph of $y = f(x)$, with all its stationary points, is shown in ***Diagram 2*** below. ***Diagram 2*** applies to **Question 19** and **Question 20**. The maximal domain of f is $\mathbb{R}$.

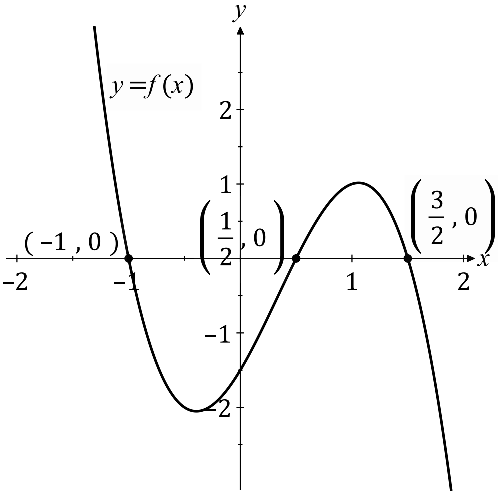


Diagram 2

Question 19

Given e is Euler's number, what is the implied domain of $\log_e(f(x))$, where the graph of $y = f(x)$ is shown in ***Diagram 2*** above?

A. $(-\infty, -1) \cup \left(\frac{1}{2}, \frac{3}{2}\right)$

B. $(-\infty, -1] \cup \left[\frac{1}{2}, \frac{3}{2}\right]$

C. $\mathbb{R}$

D. $\left[-1, \frac{1}{2}\right] \cup \left[\frac{3}{2}, \infty\right)$

Question 20

Given e is Euler's number and $x \in \mathbb{R}$, how many stationary points does the graph of $y = f(e^x)$ have, where the graph of $y = f(x)$ is shown in ***Diagram 2*** above?

A. 0

B. 1

C. 2

D. 3

END OF SECTION A

Section B

Instructions

- Answer all questions in the spaces provided.
- Write your responses in English.
- In questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Question 1 (10 marks)

A piece of wire is 100 cm long. Some of the wire is to be used to make a circle of radius r cm. the remaining wire is used to make an equilateral triangle of side length x cm. The formula for circle circumference is $2\pi r$ and the formula for circle area is πr^2.

a. By first writing an equation involving r and x and 100, express r in terms of x.

2 marks

b. State the area of the equilateral triangle in terms of x.

1 mark

c. Using your answers from parts a and b, show that the combined area of the circle and the equilateral triangle is given by

$$A(x) = \frac{1}{4}\left(\sqrt{3}x^2 + \frac{(100-3x)^2}{\pi}\right).$$

2 marks

The 100 cm of wire could be used to make one shape only (either the circle or the equilateral triangle), or it could be used to make both shapes.

d. State a sensible domain for the function A seen in part c. Give reasons.

2 marks

e. State the minimum combined area in cm^2 and its associated x-value, both correct to one decimal place.

1 mark

f. When the combined area is at its maximum, what is the area of the equilateral triangle in cm^2? Give reasons.

2 marks

Question 2 (9 marks)

Tom wants to design a game of chance using dice. The player pays $\$x$ to enter the game. The player repeatedly rolls an unbiased die until they roll a one, upon which the game ends. The player receives $1 for each roll of the die. For instance, if a player rolls two, six, five, two, and one, then the game ends, and because the player rolled five times, they receive a total of $5.
If a player rolls seven times and they still do not get a one on the die, then the game ends and the player receives $0.
On any die, the smallest face value is always one, and face values increase by 1.

a. Tom first considers the game with an unbiased six-sided die.
Let D represent the dollar amount that the player receives at the end of the game.

i. Find $\Pr(D = 0)$, correct to two decimal places.

1 mark

If $D > 0$, then the probability of the player receiving $\$D$ can be modelled by

$$\Pr(D = d) = \left(\frac{5}{6}\right)^{d-1} \times \left(\frac{1}{6}\right)$$

where $d \in \{1,2,3,4,5,6,7\}$.

ii. Complete the following probability distribution table, correct to two decimal places.

d	0	1	2	3	4	5	6	7
$\Pr(D = d)$								

1 mark

iii. Find $\text{E}(D)$.

1 mark

iv. If Tom wants to earn a positive profit in the long run, what is the minimum amount of dollars that he should charge each player to enter the game? Give your answer to the nearest cent.

1 mark

b. Tom then considers the game with an unbiased n-sided die, where $n \in \mathbb{N} \setminus \{1\}$.
Let D_n represent the dollar amount that the player receives at the end of the game.
If $D_n > 0$, then the probability of the player receiving $\$D_n$ can be modelled by

$$\Pr(D_n = d) = \left(\frac{n-1}{n}\right)^{d-1} \times \left(\frac{1}{n}\right)$$

where $d \in \{1,2,3,4,5,6,7\}$.

i. If a ten-sided die is used, find the probability that a player receives \$2 or \$3 at the end of the game, correct to two decimal places.

1 mark

ii. Find $\Pr(D_n = 0)$ in terms of n.

1 mark

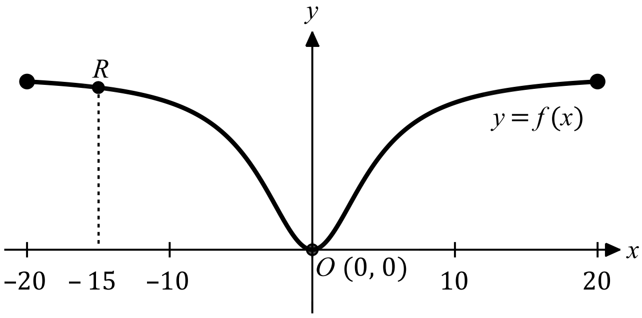
iii. In a game with a n-sided die, the probability that the player receives \$1 or \$2 at the end of the game is $\frac{39}{400}$. Find the value of n.

2 marks

iv. If Tom wants to exactly break even (make \$0 profit) in the long run, find the amount of dollars that he should charge each player to enter the game, in terms of n.

1 mark

Question 3 (11 marks)
A wheeled autonomous robot drives over land to investigate locations with hazardous substances. The diagram below shows a cross-section of a valley with a hazardous substance located at point O (the lowest point of the valley), and the robot is located at point R.



The valley can be modelled by $y = f(x)$, where

$$f: [-20,20] \rightarrow \mathbb{R}, f(x) = \frac{8x^2}{x^2 + 20}$$

Note that the x-coordinate of point R is -15 and point O has coordinates $(0, 0)$.

a. State the rule for $f'(x)$.

1 mark

The robot cannot drive on land that has a slope of less than -1 or a slope of greater than 1.

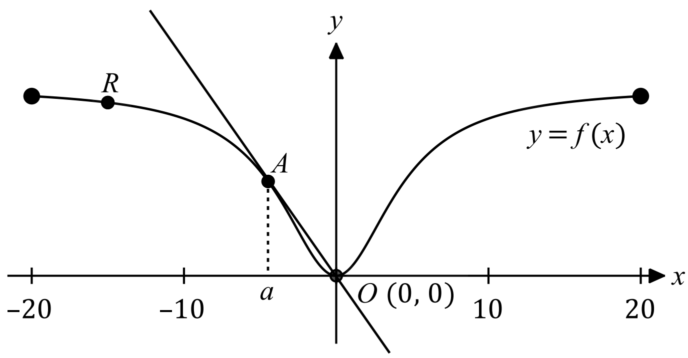
b. State the x-values where the valley is the steepest, and hence determine if the robot is able to travel from point R to point O.

2 marks

c. Find the x-value of the furthest point to which the robot can travel from point R towards the direction of point O, correct to two decimal places.

2 marks

d. Travelling from point R to the furthest possible point is too risky, so the robot stops at a point A, where the x-coordinate is a, and takes a photo of the hazardous substance at point O. The line OA is a tangent to the graph of $y = f(x)$. The situation is illustrated in the diagram shown below. Find the value of a.



3 marks

e. A different valley can be modelled by a function that results from a dilation of factor k from the x-axis from the graph of $y = f(x)$, where $k > 0$. In this different valley, the robot is able to travel throughout the valley. Find all possible values of k.

3 marks

Question 4 (20 marks)
In a farm, there are 600 male sheep and $12\ 000$ female sheep. The weights of the male sheep are normally distributed with a mean of 76.2 kg and a standard deviation of 6.8 kg.

a. Within all the male sheep, 15 of them each weigh more than x kg.

i. Find the value of x, correct to one decimal place.

1 mark

The weights of female sheep are also normally distributed but have a smaller mean and a smaller standard deviation than that of the weights of male sheep.

ii. Using standardisation and the z score, explain whether or not it is possible to have 300 female sheep each weighing more than x kg, where x is the value found in question i above.

3 marks

b. A randomly selected male sheep is known to weigh more than 80 kg.

i. Find the probability that the randomly selected male sheep weighs more than 88 kg, correct to four decimal places.

2 marks

ii. A farmer randomly selects 12 male sheep, with replacement, where each of the male sheep selected is known to weigh more than 80 kg. Find the probability that more than one of these 12 male sheep weighs more than 88 kg, correct to two decimal places.

1 mark

iii. In a group of 12 male sheep, all the sheep are known to weigh more than 80 kg. Three of them weigh more than 88 kg, and nine of them weigh less than 88 kg. A farmer randomly selects 5 male sheep from this group of 12, without replacement. Find the probability that in the 5 male sheep selected, one or two of them weigh more than 88 kg, correct to two decimal places.

2 marks

c. A researcher wishes to estimate p, the proportion of male sheep in the whole flock of 600 male sheep that weigh between 70 kg and 82 kg.

i. Using normal distribution with a mean of 76.2 kg and a standard deviation of 6.8 kg, state the value of the population proportion p, correct to four decimal places.

1 mark

The researcher selects a random sample of 400 male sheep from the flock. In this sample, 269 sheep weigh between 70 kg and 82 kg.

ii. State $\hat{p}$, the sample proportion, as a decimal number.

1 mark

iii. State the 95% confidence interval for p, correct to four decimal places.

1 mark

iv. The farmer thinks it is curious that the value of p calculated in question c.i. above is not inside the 95% confidence interval calculated in question c.iii. above. Explain why or how this could occur.

1 mark

d. It is known that in the whole flock of $12\ 000$ female sheep, the probability of randomly selecting a female sheep that weighs more than 68 kg is 0.3252.

i. The farmer randomly selects a number of female sheep from the whole flock, with replacement. What is the minimum number of female sheep that the farmer needs to randomly select such that the probability of selecting more than 5 female sheep that weigh more than 68 kg is at least 0.3557?

2 marks

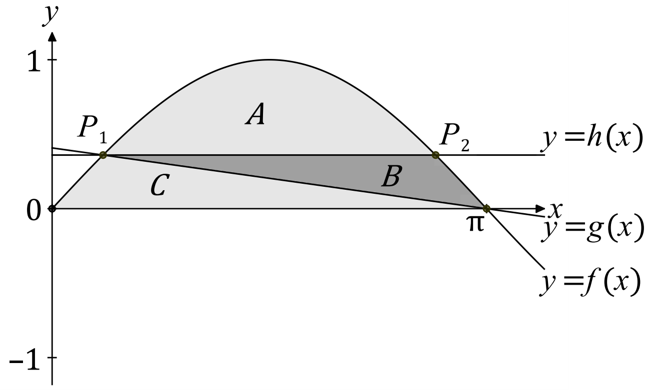
ii. In a group of 12 female sheep, n of them weigh more than 68 kg, and the rest weigh less than 68 kg. A farmer randomly selects 5 female sheep from this group of 12, without replacement. The probability that in the 5 female sheep selected, one or two of them weigh more than 68 kg is 0.6629, correct to four decimal places. Find the value of n.

2 marks

iii. A researcher wishes to estimate the proportion of female sheep in the whole flock of $12\ 000$ female sheep that weigh more than 68 kg. The researcher selects a random sample of n female sheep, and they calculate a 92% confidence interval close to $(0.3203, 0.3400)$. Find the value of n.

3 marks

Question 5 (10 marks)
Graphs of $y = f(x)$, $y = g(x)$, and $y = h(x)$ are shown below.
The graph of $y = h(x)$ is a horizontal line, function g is linear, and $f(x) = \sin(x)$.
The point P_1 is the intersection among all three graphs.
The point P_2 is the intersection between $y = f(x)$ and $y = h(x)$.
The point $(\pi, 0)$ is the intersection between $y = f(x)$ and $y = g(x)$.
The x-coordinates of P_1 are P_2 are denoted by x_1 and x_2 respectively.
The letters A, B, and C denote the bounded regions as shaded in the diagram.



a. Assume that the area of bounded region A is 1 square unit.

i. By first setting up an integral that involves the number $\frac{\pi}{2}$, find the value of x_1, correct to four decimal places.

2 marks

ii. Hence, show that $h(x) = 0.3600$ when corrected to four decimal places.

1 mark

iii. Hence, find the area of the bounded region B, correct to two decimal places.

3 marks

b. Let $h(x) = a$, where $0 \leq a \leq 1$. The area of bounded region A is unknown.

i. State the value of a for which the area of the bounded region B is at its minimum.

1 mark

ii. Find the value of a for which the area of the bounded region B is at its maximum, correct to four decimal places.

3 marks

END OF QUESTION AND ANSWER BOOK